MapReduce Algorithms

Quick Review of MapReduce
The All-Pairs Problem
The Theory of MapReduce Algorithms
Some-Pairs Problems

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What Does MapReduce Give You?

1. Easy parallel programming.
2. Invisible management of hardware and software failures.
3. Easy management of very-large-scale data.

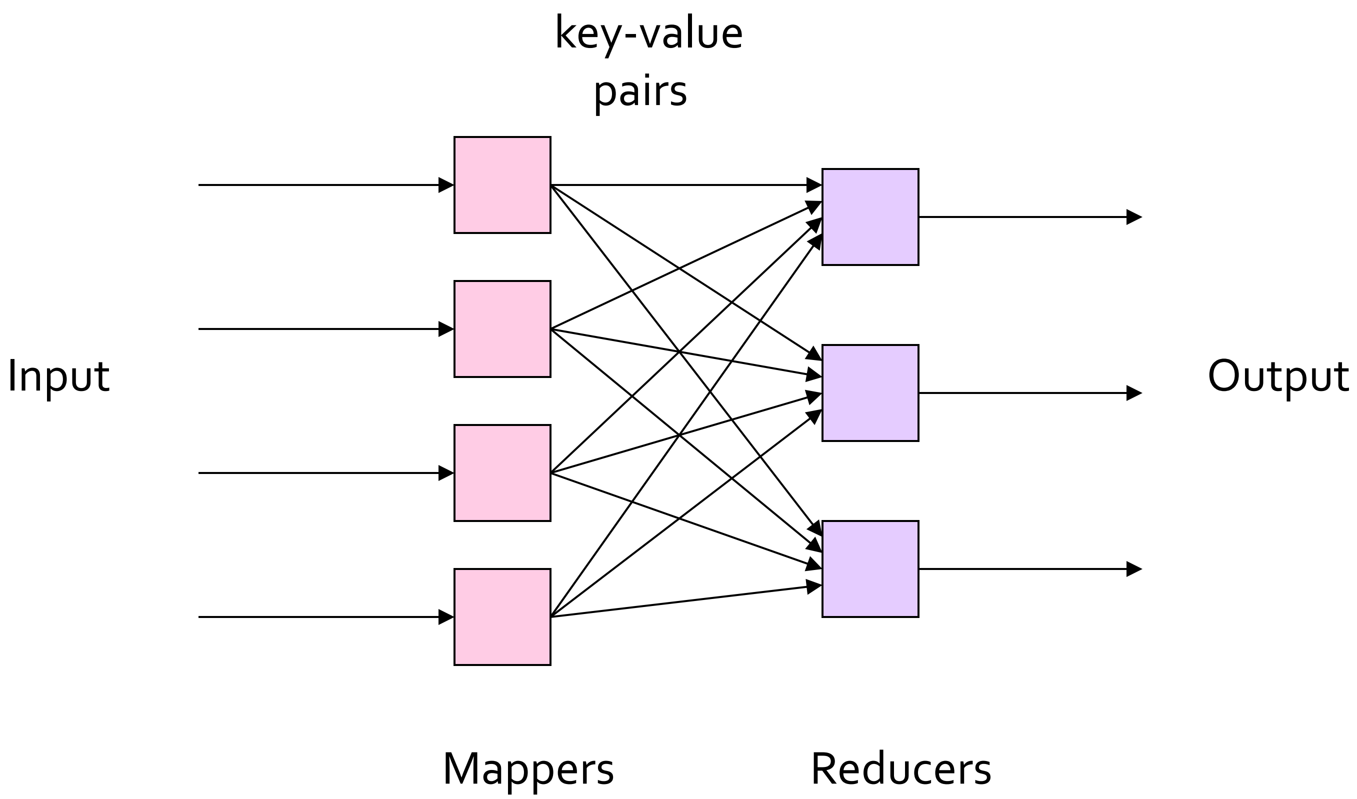
MapReduce in a Nutshell

- A MapReduce job starts with a collection of inputs of a single type.
- Apply a user-written *Map function* to each input, in parallel.
  - *Mapper* = application of the Map function to a single input.
    - Usually many mappers are grouped into a *Map Task*.
- The output of the Map function is a set of 0, 1, or more *key-value pairs*.
- The system sorts all the key-value pairs by key, forming key-(list of values) pairs.

In a Nutshell – (2)

- Another user-written function, the *Reduce function*, is applied to each key-(list of values).
  - Application of the Reduce function to one key and its list of values is a *reducer*.
    - Often, many reducers are grouped into a *Reduce Task*.
- Each reducer produces some output, and the output of the entire job is the union of what is produced by each reducer.

MapReduce Pattern



Example: Word Count

- We have a large file of documents, which are sequences of words.
- Count the number of times each distinct word appears in the file.

Word Count Using MapReduce

```
map(key, value):
// key: document ID; value: text of document
    FOR (each word w in value)
        emit(w, 1);

reduce(key, value-list):
// key: a word; value-list: a list of integers
        result = 0;
        FOR (each integer v on value-list)
                result += v;
        emit(result);
```

Expect to be all 1's, but "combiners" allow local summing of integers with the same key before passing to reducers.

Coping With Failures

- MapReduce is designed to deal with compute nodes failing to execute a Map task or Reduce task.
- Re-execute failed tasks, not whole jobs.
- Key point: MapReduce tasks have the *blocking property*: no output is used until task is complete.
- Thus, we can restart a Map task that failed without fear that a Reduce task has already used some output of the failed Map task.

Cost of a MapReduce Algorithm

1. Execution time of the mappers and reducers.
2. Communication cost of transmitting the output of the mappers to the location of the proper reducer.
   - Usually, many compute nodes handle both sorts of tasks in parallel, so there is little chance that the source and destination of a key-value pair are the same.
   - Often, communication cost dominates.

The All-Pairs Problem

Motivation: Drug Interactions
A Failed Attempt
Lowering the Communication

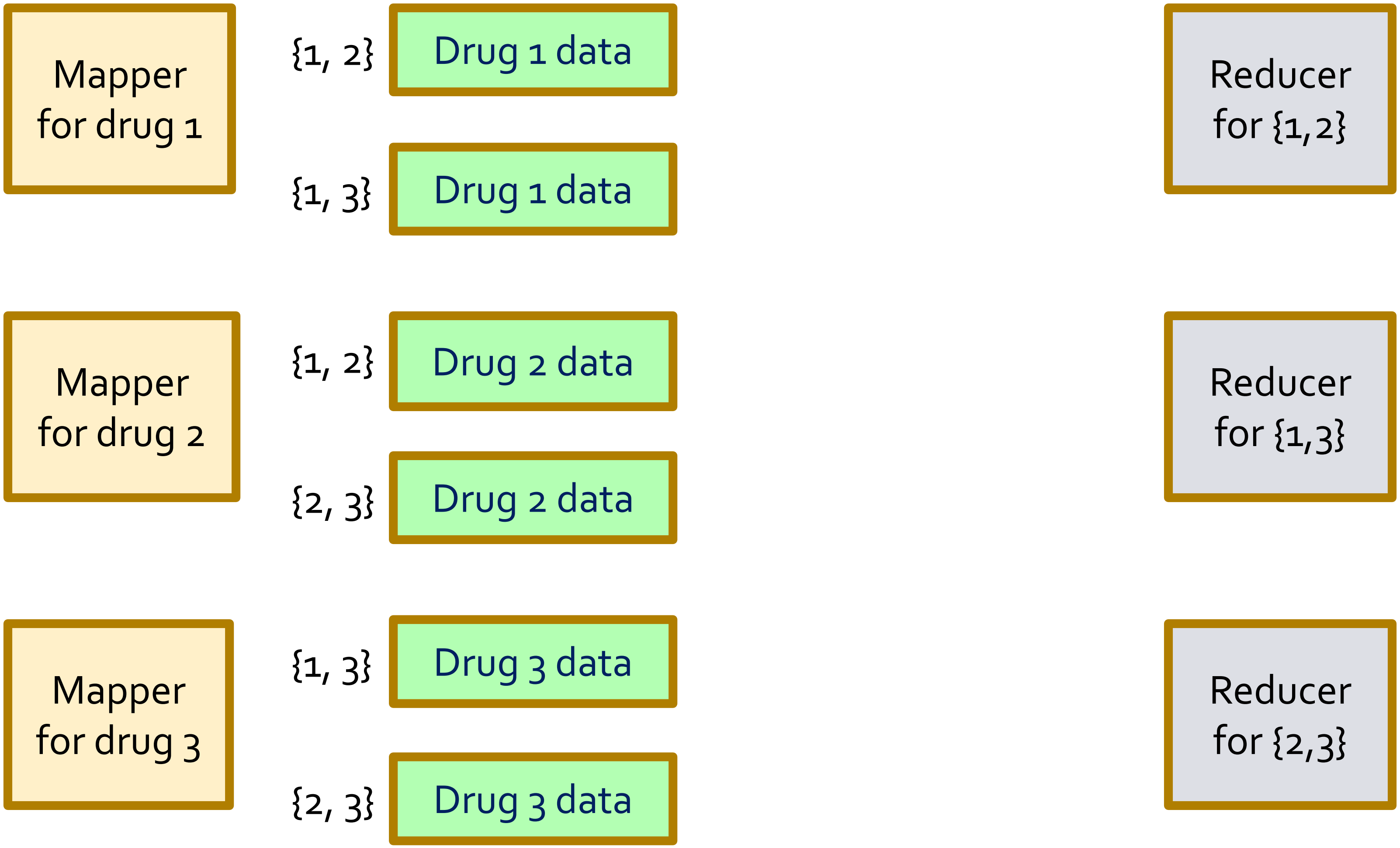
The Drug-Interaction Problem

- A real story from Stanford's CS341 data-mining project class.
- Data consisted of records for 3000 drugs.
  - List of patients taking, dates, diagnoses.
  - About 1M of data per drug.
- Problem was to find drug interactions.
  - Example: two drugs that when taken together increase the risk of heart attack.
- Must examine each pair of drugs and compare their data.

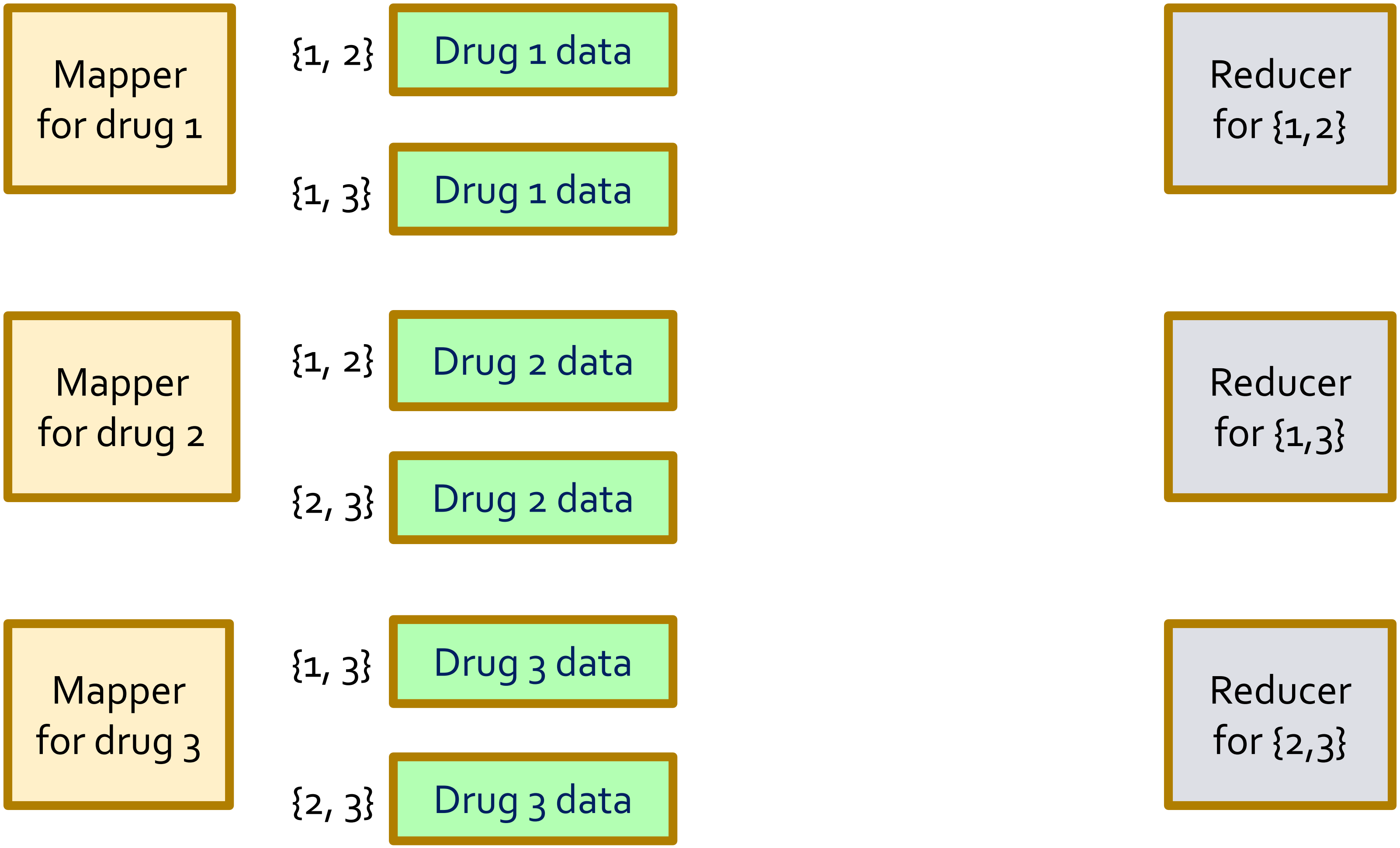
Initial Map-Reduce Algorithm

- The first attempt used the following plan:
  - Key = set of two drugs $\{i, j\}$.
  - Value = the record for one of these drugs.
- Given drug i and its record R_i, the mapper generates all key-value pairs $(\{i, j\}, R_i)$, where j is any other drug besides i.
- Each reducer receives its key and a list of the two records for that pair: $(\{i, j\}, [R_i, R_j])$.

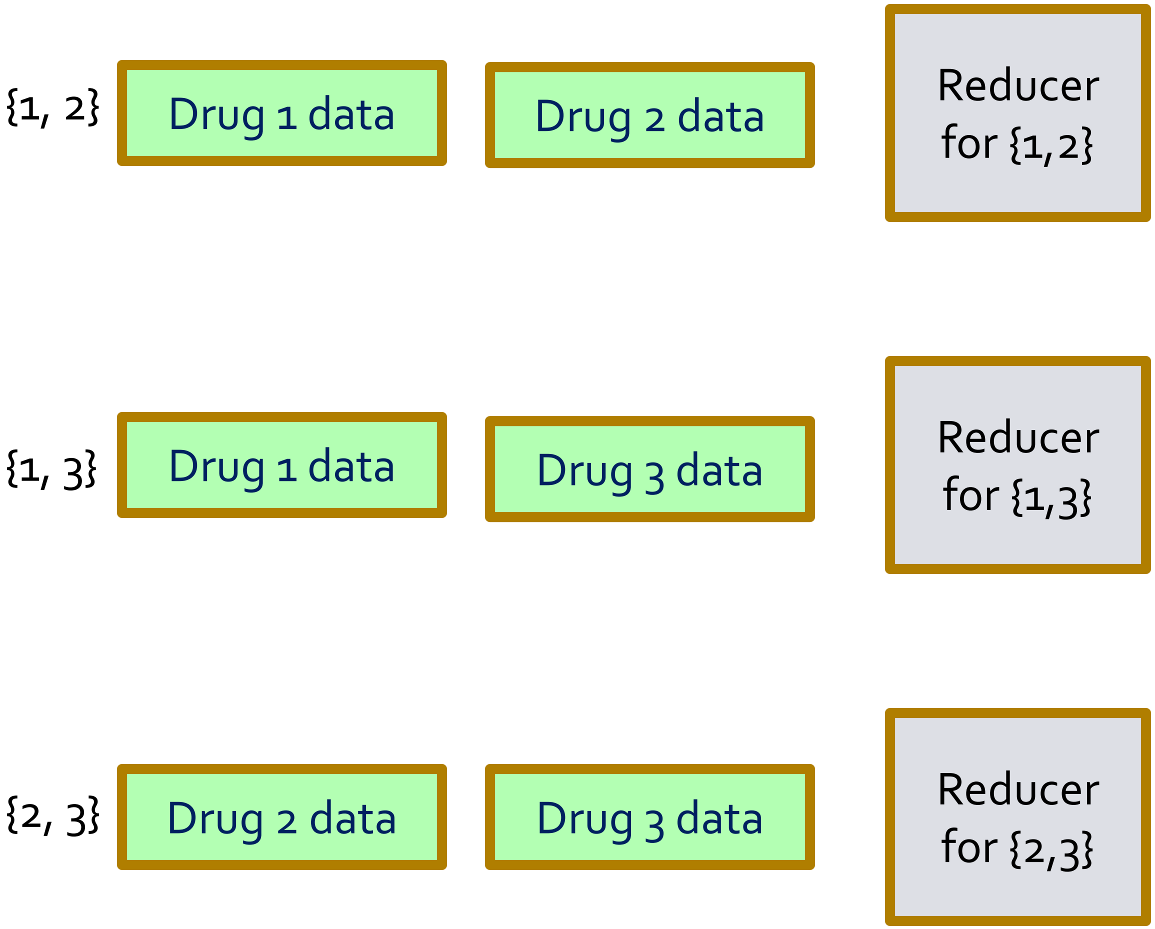
Example: Three Drugs



Example: Three Drugs



Example: Three Drugs



What Went Wrong?

- 3000 drugs
- times 2999 key-value pairs per drug
- times 1,000,000 bytes per key-value pair
- = 9 terabytes communicated over a 1Gb Ethernet
- = 90,000 seconds of network use.

The Improved Algorithm

- The team grouped the drugs into 30 groups of 100 drugs each.
  - Say G_1 = drugs 1-100, G_2 = drugs 101-200,..., G_{30} = drugs 2901-3000.
  - Let g(i) = the number of the group into which drug *i* goes.

The Map Function

- A key is a set of two group numbers.
- The mapper for drug *i* produces 29 key-value pairs.
  - Each key is the set containing g(i) and one of the other group numbers.
  - The value is a pair consisting of the drug number *i* and the megabyte-long record for drug *i*.

The Reduce Function

- The reducer for pair of groups {*m*, *n*} gets that key and a list of 200 drug records – the drugs belonging to groups *m* and *n*.
- Its job is to compare each record from group *m* with each record from group *n*.
  - Special case: also compare records in group *n* with each other, if $m = n+1$ or if $n = 30$ and $m = 1$.
- Notice each pair of records is compared at exactly one reducer, so the total computation is not increased.

The New Communication Cost

- The big difference is in the communication requirement.
- Now, each of 3000 drugs' 1MB records is replicated 29 times.
  - Communication cost = 87GB, vs. 9TB.

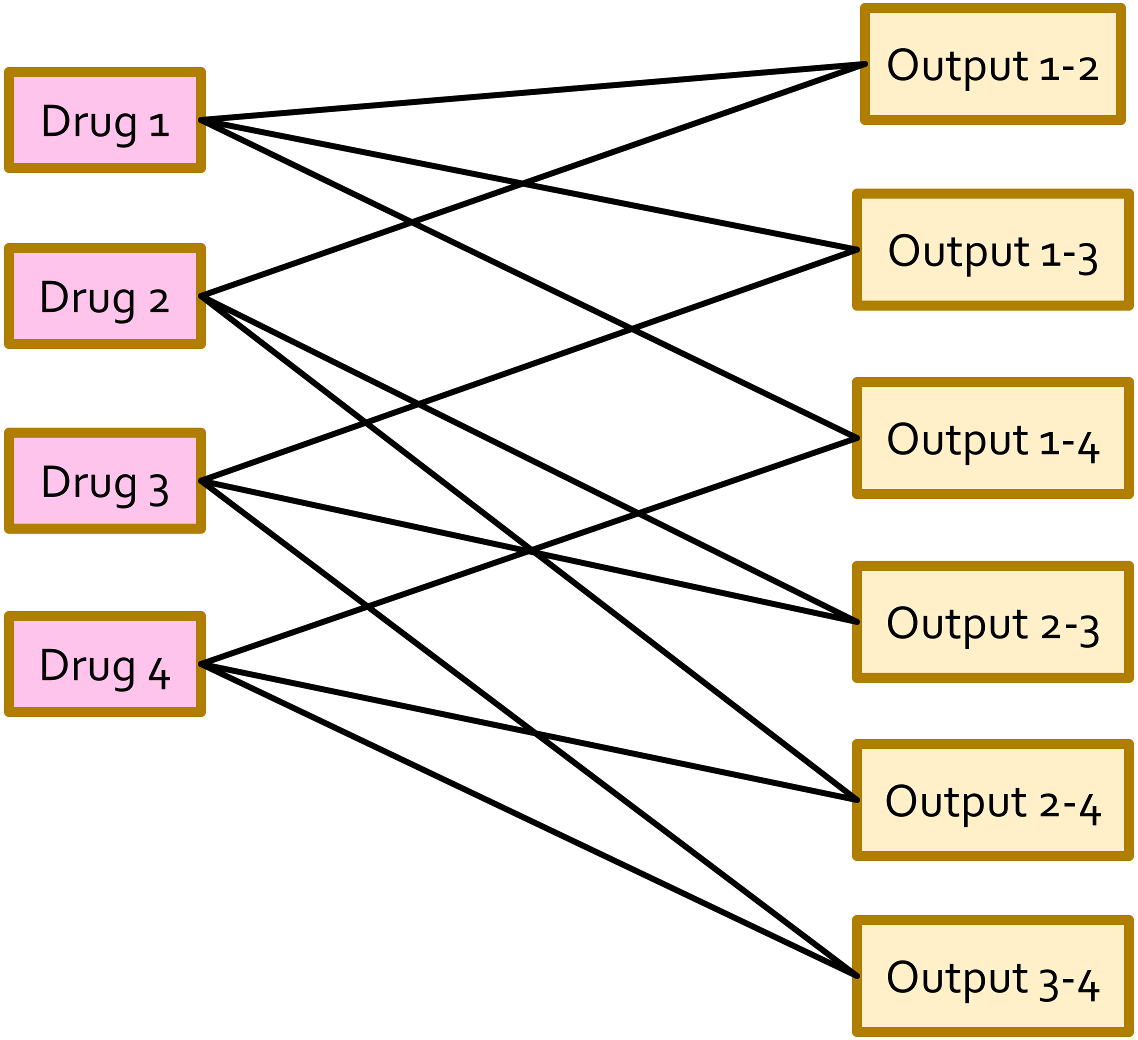
Outline of the Theory

- Work due to: Foto Afrati, Anish Das Sarma, Semih Salihoglu, U
- Reducer Size
- Replication Rate
- Mapping Schemas

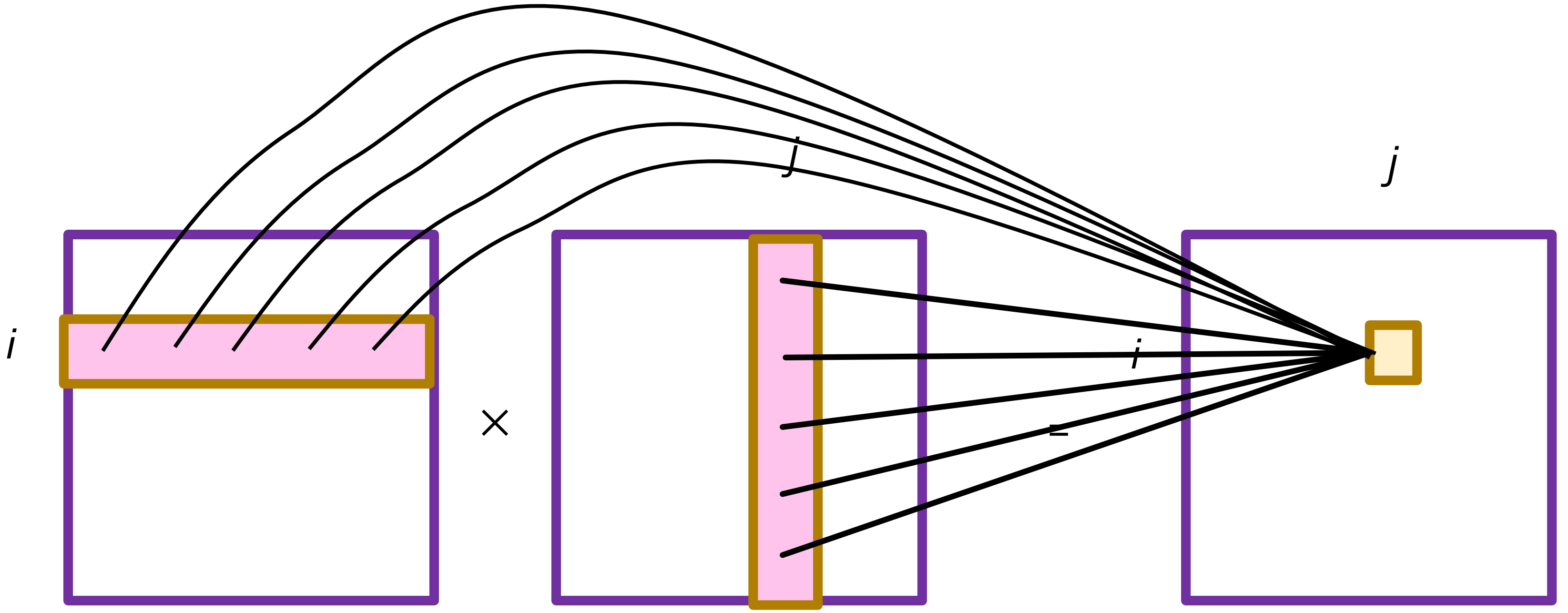
A Model for Map-Reduce Problems

1. A set of *inputs*.
   - Example: the drug records.
2. A set of *outputs*.
   - Example: one output for each pair of drugs, telling whether a statistically significant interaction was detected.
3. A many-many relationship between each output and the inputs needed to compute it.
   - Example: The output for the pair of drugs {*i*, *j*} is related to inputs *i* and *j*.

Example: Drug Inputs/Outputs



Example: Matrix Multiplication



Reducer Size

- *Reducer size*, denoted q, is the maximum number of inputs that a given reducer can have.
  - I.e., the length of the value list.
- Limit might be based on how many inputs can be handled in main memory.
- Or: make q low to force lots of parallelism.

Replication Rate

- The average number of key-value pairs created by each mapper is the *replication rate*.
  - Denoted r.
- Represents the communication cost per input.

Example: Drug Interaction

- Suppose we use g groups and d drugs.
- A reducer needs two groups, so $q = 2d/g$.
- Each of the d inputs is sent to g-1 reducers, or approximately $r = g$.
- Replace g by r in $q = 2d/g$ to get $r = 2d/q$.

Tradeoff!
The bigger the reducers,
the less communication.

Upper and Lower Bounds on r

- What we did gives an upper bound on r as a function of q.
- A solid investigation of MapReduce algorithms for a problem includes lower bounds.
  - Proofs that you cannot have lower r for a given q.

Proofs Need Mapping Schemas

- A *mapping schema* for a problem and a reducer size q is an assignment of inputs to sets of reducers, with two conditions:
  1. No reducer is assigned more than q inputs.
  2. For every output, there is some reducer that receives all of the inputs associated with that output.
     - Say the reducer *covers* the output.
     - If some output is not covered, we can't compute that output.

Mapping Schemas – (2)

- Every MapReduce algorithm has a mapping schema.
- The requirement that there be a mapping schema is what distinguishes MapReduce algorithms from general parallel algorithms.

Example: Drug Interactions

- d drugs, reducer size q.
- Each drug has to meet each of the d-1 other drugs at some reducer.
- If a drug is sent to a reducer, then at most q-1 other drugs are there.
- Thus, each drug is sent to at least $\lceil(d-1)/(q-1)\rceil$ reducers, and $r \geq \lceil(d-1)/(q-1)\rceil$.
  - Or approximately $r \geq d/q$.
- Half the r from the algorithm we described.
- Better algorithm gives $r = d/q + 1$, so lower bound is actually tight.

Some-Pairs Problems

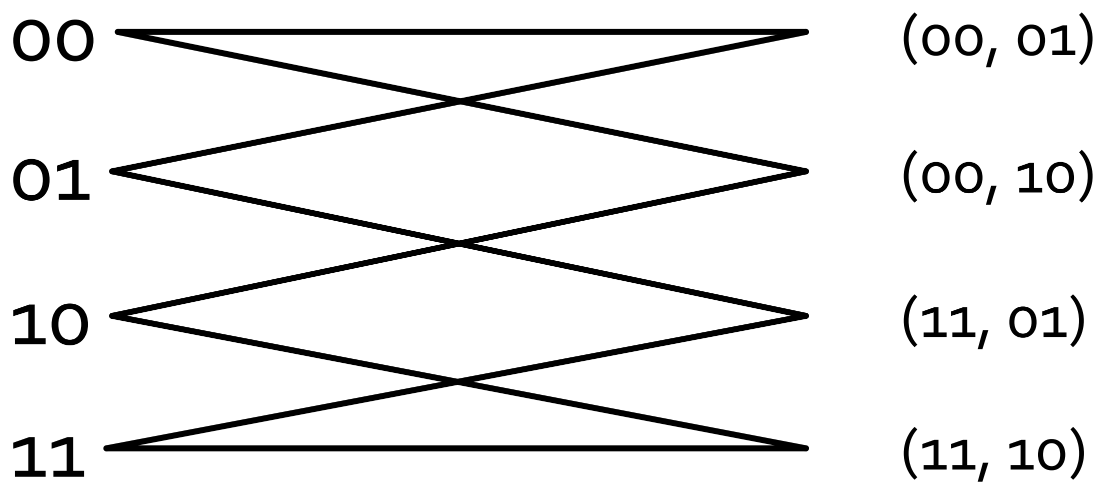
Work due to: Jonathan Ullman, U
Example: Hamming Distance
Two Obvious Approaches
Lower Bound

Some-Pairs Problems

- Outputs = a subset of the pairs of inputs.
- Example: HD1.
  - Inputs = bit strings of length b.
  - Outputs = all pairs of inputs that are at Hamming distance 1.
    - *Hamming distance* = number of positions in which strings differ.
- Known upper and lower bound: $r = b/\log_2 q$.

Example: HD1

Inputs



Outputs

Note pairs (00, 11) and (01, 10) are NOT outputs.

General-Purpose Algorithms

- Some particular some-pairs problems have really good solutions.
  - Example: HD1
- But we're looking for a single algorithm that solves any some-pairs problem and takes advantage of the fact that not all pairs of inputs are outputs.

Obvious Algorithm #1

- Let there be n inputs and m outputs.
- Assume all pairs are outputs, and use the all-pairs solution.
- Gives us $r = n/q$, independent of m.

Obvious Algorithm #2

- For each of the m outputs, create a reducer for only the two inputs associated with that output.
- Requires only $q = 2$.
- Gives us replication rate $r = 2m/n$.

Optimality of the Obvious

- Theorem: For any n, m, and q, there is a some-pairs problem whose replication rate is "almost" as large as min(m/n, n/q).
- More precisely, $r \geq \min\left(\varepsilon m/n, (n/q)^{1-\varepsilon}\right)$ for any $\varepsilon > 0$.
- Note: Most common problems will have better solutions; this lower bound only limits what a *general-purpose* algorithm can do.

Summary

- MapReduce is an important tool for failure-resistant parallel computation.
- The theory of algorithm design for MapReduce is in its infancy.
  - Involves the tradeoff between how much work to assign to a reducer and the amount of communication needed.
  - Many open questions remain, e.g., “Hamming-distance 2.”